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HEAT EXCHANGE IN LAMINAR FLOW THROUGH PLANAR AND QUASIPLANAR CHANNELS OF VARIABLE CROSS SECTION

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Formulas are derived for both the local and averaged over the surface Nusselt number. Experimental results are presented and the limits of applicability of the calculated formulas are indicated.

We denote channels whose walls lie in parallel planes as planar, while channels with a constant distance between the walls and with a radius of curvature of the channel wall generatrix much greater than this distance will be referred to as quasiplanar. The analytic solution of the problem of heat exchange in the laminar flow of a liquid through plane channels has been considered in many studies (for example, [1]). However, the available analytic studies are dedicated to heat exchange in planar channels of constant cross section, in which the mean motion velocity is constant. Experimental data for planar channels of variable cross section were presented in [2, 3], but these are of a partial character.

We will consider the analytic solution of the problem of heat exchange for a laminar liquid flow in quasiplanar channels (Fig. 1a, c). A schematic diagram of a portion of the channel is shown in Fig. 1d. In formulating the problem we make the following assumptions, in addition to those usually made ([1], p. 76): a) the liquid flow and temperature field have azimuthal symmetry; b) the channel width is a fixed function of the longitudinal coordinate (by channel width we understand the length of the line perpendicular to the flow lines and located in a plane parallel to the channel walls); c) the velocity profile over the channel height at section x is parabolic:

$$\omega_x = \frac{3}{2}\bar{\omega}\left(1-\frac{y^2}{h^2}\right) = \frac{3}{2}\,\frac{G_m}{\delta f(x)\,\rho}\left(1-\frac{y^2}{h^2}\right). \qquad (1)$$

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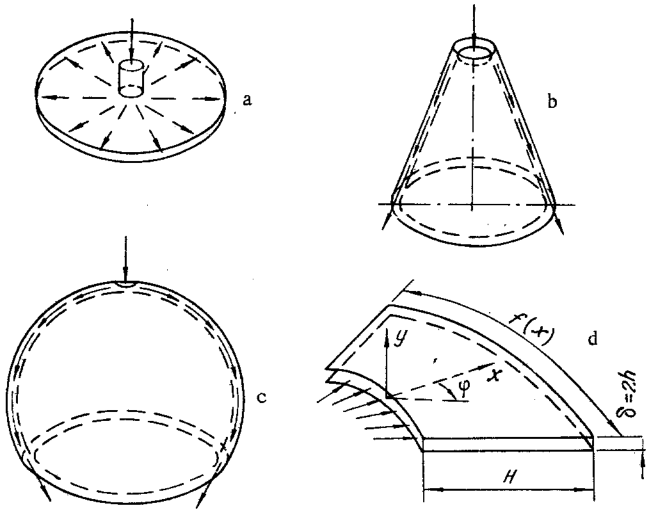

Fig. 1. Models of planar and quasiplanar channels on surface of body of rotation.

In analogy to [1], we may write

$$\frac{3}{2}\frac{G_m\delta}{4a\rho}\frac{(1-Y^2)}{f(x)}\frac{\partial\theta}{\partial x}=\frac{\partial^2\theta}{\partial Y^2}, \tag{2}$$

where

$$\theta=\frac{t-t_w}{t_0-t_w};\quad Y=\frac{y}{h};\quad t_w=\text{const}.$$

The boundary conditions are as follows:

$$\text{at}\quad x=0 \text{ and } -1<Y<1,\quad \theta=1; \tag{3a}$$

$$\text{at}\quad x\geqslant 0\quad\text{and}\quad Y=0,\ \frac{\partial\theta}{\partial Y}=0; \tag{3b}$$

$$\text{at}\quad x\geqslant 0\quad\text{and}\quad Y=\pm 1,\quad \theta=0. \tag{3c}$$

In solving the problem by the method of separation of variables, we obtain

$$\theta=\sum_{n=0}^{\infty}C_n\psi_n(Y)\exp\left(-\frac{8}{3}\varepsilon_n^2\frac{a\rho}{G_m}\frac{S(x)}{\delta}\right), \tag{4}$$

where $\psi_n(Y)$, ε_n are eigenfunctions and eigennumbers of the problem,

$$C_n=-\frac{2}{\varepsilon_n\left(\frac{\partial\psi_n}{\partial\varepsilon}\right)\Big|_{\varepsilon=\varepsilon_n;\,Y=1}}. \tag{5}$$

We introduce the generalized modified Peclet criterion

$$\mathrm{Pe}_x^*=\frac{G_m\delta}{a\rho S(x)};\quad \mathrm{Pe}^*=\frac{G_m\delta}{a\rho S(H)}. \tag{6}$$

Then we may write

$$\theta=\sum_{n=0}^{\infty}C_n\psi_n(Y)\exp\left(-\frac{8}{3}\varepsilon_n^2\frac{1}{\mathrm{Pe}_x^*}\right). \tag{7}$$

We define the mean mass temperature at section x:

$$\bar\theta=\frac{t-t_w}{t_0-t_w}=3\sum_{n=0}^{\infty}B_n\frac{1}{\varepsilon_n^2}\exp\left(-\frac{8}{3}\varepsilon_n^2\frac{1}{\mathrm{Pe}_x^*}\right), \tag{8}$$

where

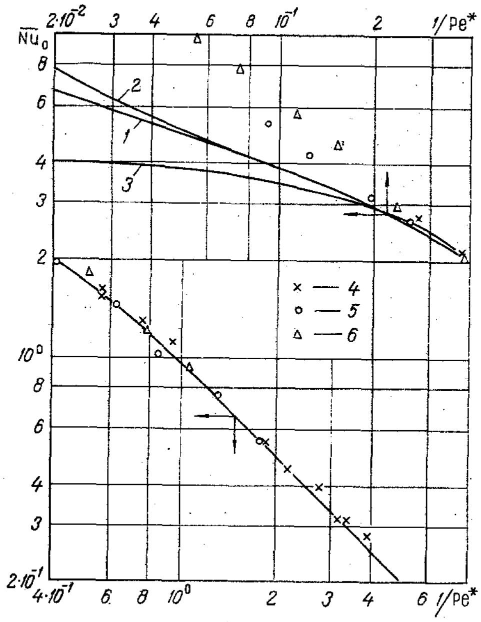

Fig. 2. Mean integral Nusselt criterion referred to temperature difference ($t_W - t_0$) over the surface $\overline{Nu}_0$ versus Pe*: 1-3) from Eqs. (17), (18), and (25); 4-6) experimental data for $\delta = 1$, 3, and 5 mm, respectively.

$$B_n = \frac{1}{2} C_n \left(\frac{\partial \psi_n}{\partial Y}\right)\Big|_{Y=1}. \tag{9}$$

The local Nusselt number, referred to the local temperature difference ($t_W - \bar{t}$) ([1], p. 86), is as follows:

$$\mathrm{Nu} = -\frac{2}{\bar{\theta}}\left(\frac{\partial\theta}{\partial Y}\right)\Big|_{Y=1} = \frac{4\sum\limits_{n=0}^{\infty} B_n \exp\left(-\frac{8}{3}\varepsilon_n^2 \frac{1}{\mathrm{Pe}_x^*}\right)}{3\sum\limits_{n=0}^{\infty} B_n \frac{1}{\varepsilon_n^2}\exp\left(-\frac{8}{3}\varepsilon_n^2\frac{1}{\mathrm{Pe}_x^*}\right)}. \tag{10}$$

As $(1/\mathrm{Pe}_x^*) \to \infty$,

$$\mathrm{Nu} \to \mathrm{Nu}_\infty = \frac{4}{3}\varepsilon_0^2 = 3.770. \tag{11}$$

We define the mean integral heat-liberation coefficient over the surface and the corresponding Nusselt number:

$$\bar{\alpha} = \frac{1}{S(H)}\int_0^H \alpha f(x)\,dx; \quad \overline{\mathrm{Nu}} = \frac{\bar{\alpha}\delta}{\lambda}.$$

From the thermal-balance equation for element dx at $t_W = \mathrm{const}$

$$\alpha f(x)\,dx = -\frac{G_m c_p}{2}\frac{d\bar{\theta}}{\bar{\theta}}. \tag{12}$$

Hence,

$$\bar{\alpha} = -\frac{G_m c_p}{2S(H)}\int_1^{\bar{\theta}|_{x=H}} \frac{d\bar{\theta}}{\bar{\theta}} = -\frac{1}{2}\frac{G_m c_p}{S(H)}\ln\bar{\theta}|_{x=H}; \tag{13}$$

$$\overline{\mathrm{Nu}} = -\frac{1}{2}\frac{G_m\delta}{a\rho S(H)}\ln\bar{\theta}|_{x=H}. \tag{14}$$

Substituting Eq. (8) for $\bar{\theta}|_{x=H}$ into Eq. (14), we obtain

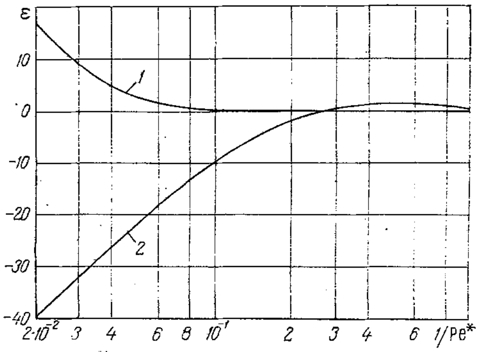

Fig. 3. Relative error ε (%) versus Pe* from Eq. (29): 1) $\varepsilon_1 = f(1/Pe^*)$; 2) $\varepsilon_2 = f(1/Pe^*)$.

$$\overline{Nu} = -\frac{1}{2} Pe^* \ln \left[3 \sum_{n=0}^{\infty} B_n \frac{1}{\varepsilon_n^2} \exp\left(-\frac{8}{3} \varepsilon_n^2 \frac{1}{Pe^*}\right)\right]. \tag{15}$$

In solving many practical problems the values of $\bar{\alpha}_0$ and $\overline{Nu}_0$ referred to the temperature difference $(t_w - t_0)$ are of interest.

By definition

$$\bar{\alpha}_0 = \frac{1}{S(H)} \int_0^H \alpha_0 f(x)\, dx; \quad \alpha_0 = \frac{q_w}{t_w - t_0} = \alpha\bar{\theta}.$$

Therefore, with consideration of Eq. (12),

$$\bar{\alpha}_0 = \frac{G_m c_p}{2S(H)} (1 - \bar{\theta}_{,x=H}); \tag{16}$$

$$\overline{Nu}_0 = \frac{\bar{\alpha}_0 \delta}{\lambda} = \frac{1}{2} Pe^* \left[1 - 3 \sum_{n=0}^{\infty} B_n \frac{1}{\varepsilon_n^2} \exp\left(-\frac{8}{3} \varepsilon_n^2 \frac{1}{Pe^*}\right)\right].$$

Equation (17), calculated with the first 11 terms of the series, is shown in Fig. 2 (curve 1). Special cases are as follows:

1. As $Pe^* \to 0$, $\overline{Nu}_0 \to (^1/_2)Pe^*$.

2. For $Pe^* < 60$ with an error less than 10% and for $Pe^* < 40$ with an error less than 3% in Eq. (17) we may consider only the first term of the series:

$$\overline{Nu}_0 = \frac{1}{2} Pe^* \left[1 - 0.91 \exp\left(-3.77 \frac{2}{Pe^*}\right)\right]. \tag{18}$$

Curve 2 of Fig. 2 corresponds to Eq. (18).

3) As $Pe^* \to \infty$, $\overline{Nu}_0 \to (^1/_2)Pe^*(1 - 3 \sum_{n=0}^{\infty} B_n(1/\varepsilon_n^2)) \approx 0.021\,Pe^*$.

4. Channel of constant width: $f(x) = B = \text{const}$. Hence, it is evident that the solutions obtained earlier, for example, in [1] and [4], are special cases of the solutions obtained now.

The solution of the heat-exchange problem in a channel of variable cross section considered here may be obtained by a simpler approximate integral method. We make the additional assumption of thermal stabilization of the flow, and in analogy to [4] approximate the temperature profile at each section by a fourth-degree polynomial:

$$\theta = a_0(x) + a_2(x) Y^2 + a_4(x) Y^4. \tag{19}$$

Substituting $Y = 1$ into Eq. (2) we obtain $\partial^2\theta/\partial Y^2|_{Y=1} = 0$ and, consequently, $a_4(x) = -(^1/_6)a_2(x)$. From condition (3c) $a_2(x) = -(^6/_5)a_0(x)$. Multiplying both sides of Eq. (2) by dY and integrating over Y in the interval [0, 1] with consideration of Eq. (19), we obtain

$$a_0(x) = C \exp\left(-\frac{140}{17} \frac{1}{Pe^*}\right). \tag{20}$$

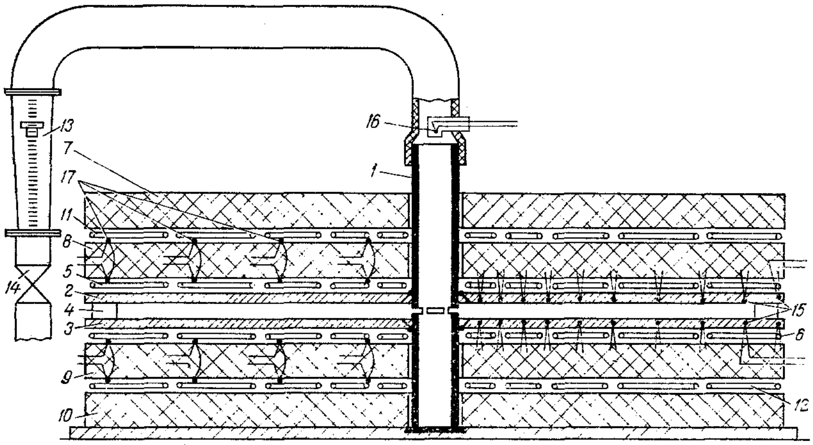

Fig. 4. Schematic diagram of experimental apparatus.

Thus,

$$\theta = C\exp\left(-\frac{140}{17}\frac{1}{\mathrm{Pe}_x^*}\right)\left[1-\frac{6}{5}\left(Y^2-\frac{1}{6}Y^4\right)\right]. \tag{21}$$

Since at $x = 0, \bar{t} = t_0$ and $S(x) = 0$, $C = (175/136)$. Then

$$\bar{\theta} = \frac{3}{2}\int_0^1 \theta(1-Y^2)\,dY = \exp\left(-\frac{140}{17}\frac{1}{\mathrm{Pe}_x^*}\right). \tag{22}$$

The local Nusselt number, referred to the local temperature difference $(t_w - \bar{t})$ for the thermally stabilized flow model considered, remains constant in any section x:

$$\mathrm{Nu}_{\mathrm{st}} = -\frac{2}{\bar{\theta}}\left(\frac{\partial\theta}{\partial Y}\right)\bigg|_{Y=1} = \frac{70}{17} \approx 4.12. \tag{23}$$

The local Nusselt number $\mathrm{Nu}_{\mathrm{st}}$ obtained by the approximate method with Eq. (23) differs by 9% from the value Nu_∞ obtained from Eq. (11) by separation of variables. The approximate equation (23) is valid for any function $t_w = t_w(x)$ if the remaining conditions are fulfilled.

Further,

$$\bar{\alpha}_0 = \frac{G_m c_p}{2S(H)}(1-\bar{\theta}|_{x=H}) = \frac{G_m c_p}{2S(H)}\left[1-\exp\left(-\mathrm{Nu}_{\mathrm{st}}\frac{2}{\mathrm{Pe}^*}\right)\right], \tag{24}$$

$$\overline{\mathrm{Nu}}_0 = \frac{1}{2}\mathrm{Pe}^*\left[1-\exp\left(-4.12\frac{2}{\mathrm{Pe}^*}\right)\right]. \tag{25}$$

Equation (25) is shown in Fig. 2 (curve 3). Figure 3 shows relative errors ε_1 and ε_2 as functions of $1/\mathrm{Pe}^*$, where

$$\varepsilon_1 = \frac{\overline{\mathrm{Nu}}_{0,2}-\overline{\mathrm{Nu}}_{0,1}}{\overline{\mathrm{Nu}}_{0,1}}100\%;\quad \varepsilon_2 = \frac{\overline{\mathrm{Nu}}_{0,3}-\overline{\mathrm{Nu}}_{0,1}}{\overline{\mathrm{Nu}}_{0,1}}100\%. \tag{26}$$

Here $\overline{\mathrm{Nu}}_{0,1}$; $\overline{\mathrm{Nu}}_{0,2}$; and $\overline{\mathrm{Nu}}_{0,3}$ are defined by Eqs. (17), (18), and (25), respectively.

For $\mathrm{Pe}^* < 20$ one may use Eq. (25) instead of Eq. (17) with an error less than 10%. Thus, the integral method may be recommended as simpler for analysis of heat-exchange processes with other boundary conditions also.

For example, for a quasiplanar channel with one wall thermally insulated and the other at constant temperature, the method of separation of variables gives

$$\overline{\mathrm{Nu}}_0 = \mathrm{Pe}^*\left[1-0{,}896\exp\left(-2.43\frac{1}{\mathrm{Pe}^*}\right)\right]; \tag{27}$$

$$\mathrm{Nu}_\infty = \frac{1}{6}\varepsilon_0^2 = 2{,}430, \tag{28}$$

while the approximate integral method gives

$$\overline{\mathrm{Nu}}_0 = \mathrm{Pe}^* \left[1 - \exp\left(-\frac{45}{17}\,\frac{1}{\mathrm{Pe}^*}\right)\right]; \tag{29}$$

$$\mathrm{Nu}_{st} = \frac{45}{17} \approx 2.65. \tag{30}$$

The error in determination of the stabilized value of the local Nusselt number from Eq. (30) as compared to Eq. (28) comprises 9%, while the error in calculation by Eq. (29) as compared to Eq. (27) does not exceed 10% for $\mathrm{Pe}^* \leq 6$.

This analytic solution of the heat-exchange problem by both methods is based on a number of assumptions, the most significant of which is assumption (c) as to the parabolicity of the velocity profile over channel height. In order to determine the limits of applicability of the models used herein an experimental apparatus, a diagram of which is shown in Fig. 4, was built and tested. The channel was formed of two aluminum disks 2 and 3, 4 mm in thickness and 300 mm in diameter. Channel height was maintained by special calibrated disks 4. Air is introduced through tube 1 (diameter 40 mm) through an orifice in its lateral surface and flows radially between the disks. The channel walls are heated by Nichrome heaters 5 and 6, each of which consists of five independently controllable concentric planar sections. To minimize heat loss to the surrounding medium, thermal insulators 7, 8, 9, and 10 and two guard heaters 11 and 12 are used. To reduce heat loss to tube 1 the inner edges of the disks are sharpened and special insulating packing is installed between them and the tube.

The cooled air supply is regulated by valve 14, and the flow rate is measured by rotameter 13 (RS-5 and RS-7). The disk temperature is monitored by 20 Chromel—Alumel thermocouples 15, while the air temperature at the input is measured by Chromel—Alumel thermocouple 16.

The power supplied to individual heater sections was selected such that the channel-wall temperature field was uniform. Guard heater power was set so that temperature was equal on the main and guard heaters, as monitored by eight differential thermocouples 17, whose junctions were located one opposite the other (a PP-63 potentiometer was used with the thermocouples).

Parameters were varied over the following ranges: $G_m = 7.53 \cdot 10^{-4}$–$1.34 \cdot 10^{-2}$ kg/sec, Q = 15–65 W, and δ = 1–5 mm.

The mean square error of the experiments was 4%. The experimental values of $\overline{\mathrm{Nu}}_{0,\mathrm{exp}}$ with corresponding values of $1/\mathrm{Pe}^*$ are shown in Fig. 2.

Several experiments were performed with the cooling air moved in the opposite direction by means of an ejection device. For $\mathrm{Pe}^* < 5$ change of flow direction does not affect the value of $\overline{\mathrm{Nu}}_0$.

At $\mathrm{Pe}^* < 5$ the maximum deviation of experimental points from the calculated curve 1 comprises not more than 10%, which lies within the limits of experimental error, but at $\mathrm{Pe}^* > 5$ both the exact (separation of variables) and approximate (integral method) solutions give a large error because of the imperfection of the model in not considering flow turbulization at the channel input and the initial hydraulic stabilization segment. At $\mathrm{Pe}^* < 5$ one can use the flow thermal stabilization assumption and use Eq. (25) for engineering calculations. Thus, the upper limit of Pe^* values at which the results obtained are valid has been determined.

In formulating the problem, among other conditions the assumption was made that it is possible to neglect longitudinal thermal conductivity. On the basis of this we will estimate the lower usable limit of Pe^* values. According to [1], we may write

$$\frac{\bar{\omega}x}{a} \geqslant \frac{1}{\varepsilon} \quad \text{or} \quad \frac{G_m x}{\rho f(x)\,\delta a} \geqslant \frac{1}{\varepsilon},$$

where ε is the admissable error.

From this we obtain a formula for the smallest possible Pe^*:

$$\mathrm{Pe}_x^* \geqslant \frac{1}{\varepsilon k}\left(\frac{\delta}{x}\right)^2, \quad k = \frac{\int_0^x f(x)\,dx}{f(x)\,x}.$$

For many particular cases the approximation $f(x) \approx a + cx^n$ is valid. Then

$$k = \frac{(n+1)\,a + cx^n}{(n+1)\,a + c\,(n+1)\,x^n}.$$

For example, for the experimental channel described above $k = 0.565$, $\delta/x|_{x=H} = 0.0385$ and if we take $\varepsilon = 0.05$, we obtain the condition

$$Pe^*|_{x=H} \geqslant 0.052.$$

NOTATION

δ, H, f(x), height, length, and width of channel; $S(x) = \int_0^x f(x)\,dx$, area of channel wall washed with liquid at distance between channel inlet and cross section x; ω_x, longitudinal velocity component; $\bar{\omega}$, mean velocity over channel section x; $h = \delta/2$, channel half-height; G_m, mass flow rate of liquid; ρ, a, c_p, λ, density, thermal diffusivity, heat capacity, and thermal conductivity of liquid; t, local liquid temperature in channel; t_w, channel wall temperature; t_0, liquid temperature before entrance into channel; $\bar{t}$, liquid mean mass temperature at section x; α, $\bar{\alpha}$, local and mean integral (over surface) heat-liberation coefficients, referenced to temperature difference $(t_w - \bar{t})$; α_0, $\bar{\alpha}_0$, local and mean integral (over surface) heat-liberation coefficients, referenced to temperature difference $(t_w - t_0)$; Nu, $\overline{Nu}$, Nu_0, Nusselt criteria for heat-liberation coefficients α, $\bar{\alpha}$, $\bar{\alpha}_0$; Pe*, generalized modified Peclet criterion; Pe, Peclet criterion; Nu_{st}, Nusselt criterion for thermally stabilized flow; q_w, density of thermal flux from wall to liquid at section x; Q, thermal flux associated with one channel wall.

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LIQUID-FILM FLOW REGIMES ON A ROTATING SURFACE

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Liquid flow on a rotating disk is analyzed and studied experimentally. Limits of the hydrodynamic regimes are established.

Liquid-film flow on rotating surfaces is employed in thermal mass-exchange devices (evaporators, driers, absorbers, etc.), reactors, and centrifuges in various technological fields. Calculation of such devices presumes a clear understanding of flow hydrodynamics, in particular, of the limits of laminar and turbulent flow.

Despite the large number of studies dedicated to study of the hydrodynamic characteristics of film flow on rotating bodies, only [1-6] indicated the hydrodynamic regime. However, in those studies there are major differences in evaluation of the effects of various parameters (wetting density Γ, angular velocity ω, and surface dimension R) on flow stability. The effect of Γ was considered in [1, 6], while [3] considered only ω and R. In determining the flow regime, Dorfman [8] considered the effects of both angular velocity and initial film thickness δ, considering the latter known. In [4], one of the first studies of film hydrodynamics on rotating surfaces, the authors use an expression for flow-regime determination which includes all the dimensionless quantities used in the works mentioned above [3, 6, 8]:

$$Re^* = Re_\Gamma^{-1} Re_r\ Re_\delta = \left(\frac{G}{2\pi R \nu \rho}\right)^{-1} \frac{\omega R^2}{\nu} \frac{\omega \delta^2}{\nu} . \tag{1}$$

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